

The effect of the Higgs boson  
on the threshold cross-section  
in  $W$ -pair production

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**Abstract**

We investigate the uncertainty induced by the Higgs-boson mass on the determination of  $m_W$  from the LEP-2 run near the threshold for  $W$ -pair production. For a light Higgs boson the Yukawa interaction between the two slowly-moving  $W$  bosons gives rise to a correction of close to 1% to the total cross-section. This corresponds to a 15 MeV shift in the deduced  $W$  mass for a Higgs-boson mass of 60 GeV. We present a simple approximation for this correction and discuss its validity.

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## 1 Introduction

One of the main experimental results from the LEP-2 collider will be an improved measurement of the  $W$  mass [1]. Within the Standard Model this mass can already be predicted from the existing accurate measurements of other precision variables, such as  $\alpha$ ,  $G_\mu$ ,  $m_Z$  and, through radiative corrections,  $m_t$  [2]. A recent fit gives  $m_W = 80.359 \pm 0.051^{+0.013}_{-0.024}$  GeV, where the first error is dominated by the error in  $m_t$  and the second one is obtained by varying the Higgs-boson mass in the interval  $60 < m_H < 1000$  GeV, with central value  $m_H = 300$  GeV [1]. As can be seen, a significant part of the uncertainty is due to the Higgs-boson mass, which also enters through radiative corrections. One expects that an accurate measurement of the  $W$ -boson mass at LEP-2, combined with an improved determination of  $m_t$  at Fermilab [3], will either narrow down the allowed range of  $m_H$ , or point to physics beyond the Standard Model.

In the initial stages of the energy upgrade of the LEP machine, the most promising approach seems to be to exploit the sharp rise of the cross-section at the threshold for  $W$ -pair production [1]. One deduces the  $W$  mass from a comparison of the measured total cross-section with a theoretical prediction. Due to the steep slope, the envisaged statistical error of 5.6% on the total cross-section (for an integrated luminosity of  $200 \text{ pb}^{-1}$ ) corresponds to a total statistical error on  $m_W$  of 95 MeV (experimental systematic errors are expected to be much smaller). Of course we would like the theoretical prediction to be much more accurate than the projected experimental error. Moreover, since one has to trust this prediction blindly, a thorough analysis of all sources of uncertainties is needed. Effects which certainly contribute at the 1% level are initial-state radiation, non-resonant ('background') diagrams, the Coulomb correction, and leading QCD corrections [4]. Initial-state radiation is normally modelled with some kind of structure-function or shower algorithm, convoluted with the lowest-order matrix element. The non-resonant contributions have been computed by various groups [5]. The Coulomb correction is a simple factor multiplying the matrix element, and in the case of  $W$ -pair production resummation is not even necessary due to the finite width of the  $W$  boson [6]. Finally, the integrated  $\mathcal{O}(\alpha_s)$  corrections are easily included. The as yet unknown initial-final and final-final state interference effects should not influence the total cross-section very much [7].

In this letter we investigate another effect, originating from a light Higgs boson. Near threshold such a light particle mediates a sizeable Yukawa interaction between the two slowly-moving  $W$  bosons. The static potential associated with this interaction is given by

$$V(r) = -\frac{m_W^2}{4\pi v^2 r} e^{-m_H r} , \quad (1)$$

with  $m_W^2/(4\pi v^2) = \alpha/(4\sin^2\theta_w)$ . The mass of the Higgs boson determines the range of the interaction, and consequently the size of the correction. For a relatively light Higgs,  $m_W\Gamma_W \lesssim m_H^2 \ll m_W^2$ , the correction to the threshold amplitude is larger than the usual  $\mathcal{O}(\alpha/\pi) = \mathcal{O}(\alpha/4\pi\sin^2\theta_w)$  by a factor  $\pi m_W/m_H$ . A Higgs boson at the current lower bound,  $m_H \approx 60$  GeV, would give rise to a correction of the order of 1%. Of course LEP-2 itself will increase this bound, but at the initial stages at which this threshold mass measurement is performed it will not have collected enough luminosity to improve on the LEP-1 direct search bound [8].

Replacing the Standard-Model Higgs boson by the CP-even neutral Higgs bosons of the Minimal Supersymmetric Standard Model (MSSM) does not change the size of this correction in a significant way. If the masses of these two Higgs bosons are degenerate one completely recovers the SM expression; if they are unequal they each give rise to a similar correction weighted by  $\cos^2(\alpha - \beta)$  and  $\sin^2(\alpha - \beta)$  respectively, with  $\tan\beta$  the standard ratio of the Higgs vacuum expectation values, and  $\alpha$  describing the mixing in the CP-even Higgs sector. Due to the absence of detectable effects at LEP-1, a light MSSM Higgs-boson implies a small coupling to the gauge bosons, and therefore one cannot obtain significantly larger effects at LEP-2 than the SM would give.

## 2 Approximation

The only radiative corrections to  $W$ -pair production that depend on the Higgs-boson mass are the  $W$  and  $Z$  self-energies, counterterms dependent on these, the  $s$ -channel vertex corrections depicted in Fig. 1, and the  $t$ -channel box of Fig. 2.

Due to screening, the contributions to the self-energies are at most logarithmic in  $m_H$  and do not contribute to the light-Higgs enhancement. Adopting the LEP-2 input-parameter scenario advocated in Ref. [4], the  $m_H$  dependence is even further reduced. This scenario involves the use of  $\alpha$ ,  $G_\mu$ , and  $m_Z$  (and the light fermion masses) as input and treats  $m_W$  as free fit parameter. Subsequently, the top-quark mass is calculated as a function of  $m_H$  and  $\alpha_s$ , using the state-of-the-art calculation of  $\Delta r$ . As such, the logarithmic  $m_H$  dependence in the self-energies has to be largely compensated by the induced variation in  $m_t$ .

The vertex corrections only contribute to  $s$ -channel invariant amplitudes, which are suppressed near threshold by a factor  $\beta \equiv [(s - s_- - s_+)^2 - 4s_+s_-]^{1/2}/s = \mathcal{O}([\Delta^2 + \Gamma_W^2]^{1/4}/\sqrt{m_W})$ , with  $s_\pm \equiv k_\pm^2$  the invariant momentum squared of the off-shell  $W^\pm$  and  $\Delta \equiv \sqrt{s} - 2m_W$ .

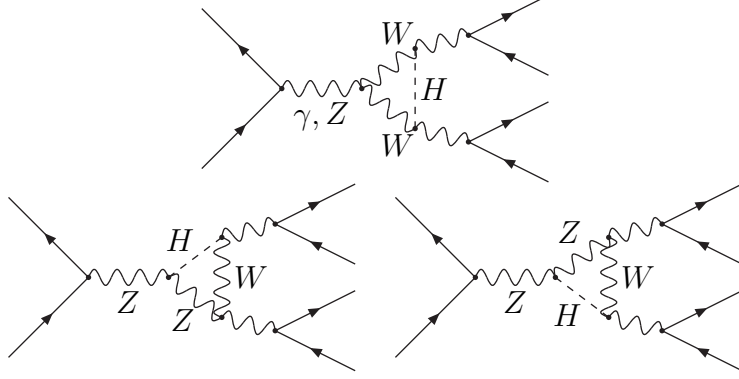


Fig. 1. The  $s$ -channel vertex diagrams that contain the Higgs boson. Only the generic diagrams involving  $W$  and  $Z$  gauge bosons are given. By replacing some of the gauge bosons in the loop by the corresponding Higgs ghosts all other diagrams can be obtained.

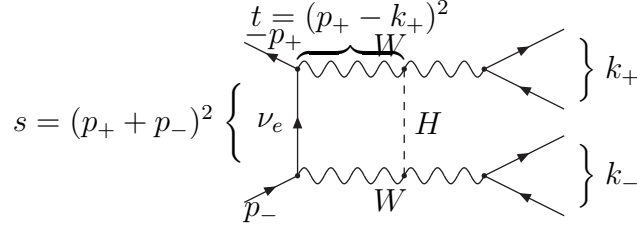


Fig. 2. The  $t$ -channel box diagram that contains the Higgs boson.

The dominant contribution will therefore come from the  $t$ -channel box and, in view of the non-relativistic, static nature of the underlying interaction, it will be proportional to the dominant lowest-order  $t$ -channel matrix element<sup>3</sup>,  $\mathcal{A}_t$ . By decomposing the amplitude of the  $t$ -channel box,  $\mathcal{A}_{\text{box}}$ , into standard matrix elements, one can derive the proportionality factor  $C_H$ . Using the notation defined in Fig. 2 we find in the Feynman gauge ( $\xi = 1$ ):

$$\begin{aligned} \mathcal{A}_{\text{box}} \approx \mathcal{A}_t \frac{\alpha}{4\pi \sin^2 \theta_w} \frac{-t m_W^2/2}{t^2 + s_+ s_- - t(s_+ + s_- - s)} \times \\ \left\{ D_0 [m_H^2(2s_+ - 2t - s) - 2\frac{m_W^2}{s}(s_+^2 - s_+ s_- + ts_- - ts_+)] \right. \\ \left. + t(s - 2s_+ + 2t) + m_W^2(2t + s_+ + s_-) \right\} \\ + C_0^{\nu W^- H} [3t + s_- - 2\frac{t}{s}(s_+ + s_- - t) + \frac{2}{s}s_+ s_-] \end{aligned}$$

<sup>3</sup> Note that at threshold the SU(2) gauge cancellation between  $s$ - and  $t$ -channel graphs does not play a role.

$$\begin{aligned}
& + C_0^{WWH} \left[ s - 3s_+ - s_- - \frac{2}{s} (ts_+ - ts_- + s_+s_- - s_+^2) \right] \\
& + C_0^{W^+\nu H} \left[ -t + s_+ - \frac{2}{s} (-2ts_+ + t^2 + s_+^2) \right] \\
& + C_0^{WW\nu} [-s - 2t + 2s_+] \Big\} \equiv \mathcal{A}_t C_H .
\end{aligned} \tag{2}$$

In this equation,  $D_0$  denotes the scalar four-point function corresponding to the box diagram, and the  $C_0$ 's the scalar three-point functions including the indicated propagators<sup>4</sup>. Superficially, the expression has a pole at the edge of phase space,  $\Delta_3 = -\frac{1}{4}s[t^2 + s_+s_- - t(s_+ + s_- - s)] = 0$ , but the numerator also vanishes there to give a finite result. Near the  $W$ -pair production threshold the matrix element corresponding to the  $t$ -channel box

$$\mathcal{A}_{\text{box}} \propto \int d^n l \frac{(l + p_- - k_-)^\mu}{[l^2 - m_H^2][(l + k_+)^2 - m_W^2][(l - k_-)^2 - m_W^2](l + p_- - k_-)^2} \tag{3}$$

can be simplified considerably by exploiting the fact that to a first approximation the two  $W$  bosons are effectively at rest and have an energy close to the beam energy [since  $|s_\pm - m_W^2| \lesssim \mathcal{O}(m_W \Gamma_W)$ ]. Combined with the symmetry of the integral under the exchange  $(p_+, k_+) \leftrightarrow (-p_-, -k_-)$  this leads to the effective replacement

$$(l + p_- - k_-)^\mu \longrightarrow (p_- - k_-)^\mu [1 + l \cdot (p_- - k_-)/t] . \tag{4}$$

Inserting all the prefactors we arrive at the following threshold approximation for  $C_H$ :

$$C_H \approx -\frac{\alpha m_W^2}{8\pi \sin^2 \theta_w} \left[ (t - m_H^2) D_0 + C_0^{WWH} - C_0^{WW\nu} \right] . \tag{5}$$

For a light Higgs ( $m_H \ll m_W$ ) the Yukawa nature of the interaction mediated by the Higgs boson dominates if the range of the Yukawa interaction,  $1/m_H$ , is shorter than the characteristic range of a Coulomb-like interaction between unstable  $W$  bosons,  $[m_W^2(\Delta^2 + \Gamma_W^2)]^{-1/4}$ . If this is the case<sup>5</sup> the leading part of Eq. (5), i.e., the part that scales with the range of the interaction, takes on the form

$$C_H \approx \frac{m_W^2}{4\pi v^2} \frac{m_W}{m_H} = \frac{\alpha}{4 \sin^2 \theta_w} \frac{m_W}{m_H} .$$

<sup>4</sup> Explicit expressions and routines for these functions may be found in Ref. [9].

<sup>5</sup> If the Higgs were to be significantly lighter,  $m_H^2 \ll m_W \sqrt{\Delta^2 + \Gamma_W^2}$ , the Yukawa interaction becomes effectively Coulomb-like [see Eq. (1)]. This would result in an enhancement of the strength of the  $W^+W^-$  Coulomb interaction according to  $\alpha \rightarrow \alpha + m_W^2/(4\pi v^2) = \alpha + \alpha/(4 \sin^2 \theta_w)$ .

$m_t$ [GeV]					
$m_H$	$m_W$ [GeV]				
[GeV]	80.10	80.18	80.26	80.34	80.42
60	119.1	133.9	148.1	161.7	174.4
300	137.3	151.6	165.3	178.3	190.7
1000	154.2	168.0	181.2	193.9	206.1

Table 1

Top-quark mass as a function of the  $W$ -boson and Higgs-boson masses.

### 3 Comparison with the full one-loop result

In order to check the accuracy of the approximations given in Eqs (2) and (5) we would like to compare with a full off-shell  $\mathcal{O}(\alpha)$  calculation. Unfortunately, this calculation is not yet available. As a first step the comparison with the full on-shell  $\mathcal{O}(\alpha)$  results [10] was performed and excellent agreement was found for all production angles of the  $W$  bosons and all  $\sqrt{s}$  up to 175 GeV (better than 0.1% in  $\sigma$ , better than 1% in  $\langle \cos \theta_W \rangle$ ). For the off-shell comparison we restrict ourselves to the factorizable parts of the full calculation and calculate these in the (gauge invariant) pole scheme [11,12], using the  $G_\mu$  renormalization scheme. For a given value of  $m_H$  we calculate the corresponding corrections and define the full Higgs-boson effect by subtracting the corrections at  $m_H = 1$  TeV. The unknown non-factorizable corrections do not depend on the Higgs-boson mass, and will cancel in the difference. The choice of subtraction point,  $m_H = 1$  TeV, is motivated by the fact that heavy Higgs bosons decouple to a large extent in our calculational scheme (based on the LEP-2 input scenario of Ref. [4]).

Before coming to the discussion of the quality of the approximations, a few technical remarks are in place. First of all it should be noted that the strict one-loop pole scheme of Ref. [11] is not well-behaved near the threshold for  $W$ -pair production. However, a recent analysis has revealed that the gauge violations of the off-shell tree-level amplitude are numerically negligible [4,13,14]. As the Higgs corrections do not contain any large threshold effects, we have combined the off-shell tree-level amplitudes [15] with the one-loop form factors in the pole scheme. In order to make the numerical results more realistic, we improve the tree-level results and Higgs-boson effects by inclusion of initial-state radiation, implemented as a structure function [16,17], and the Coulomb correction. In addition we have included the universal non-resonant graphs, i.e., the ones that contribute to all  $W$ -pair channels. In the following we will refer to these improved tree-level results

$\sigma_{\text{tot}}$ [pb]					
	$m_W$ [GeV]				
	80.10	80.18	80.26	80.34	80.42
$m_H$ [GeV]	improved Born				
any	3.941	3.768	3.599	3.435	3.274
$m_H$ [GeV]	Higgs effect: approximation Eq. (5)				
60	.0348	.0335	.0322	.0308	.0293
300	.0022	.0021	.0021	.0020	.0019
1000	.0002	.0002	.0002	.0002	.0002
$m_H$ [GeV]	Higgs effect: full result				
60	.0351	.0343	.0323	.0296	.0265
300	.0004	.0009	.0010	.0011	.0010
1000	0	0	0	0	0

Table 2

Higgs effect on the total cross-section at  $\sqrt{s} = 161$  GeV as a function of the  $W$ -boson and Higgs-boson masses.

as ‘improved Born’.

Using for the input parameters the values specified in Ref. [4], we checked the effect of the different approximations at  $\sqrt{s} = 161$  GeV for a sample of Monte Carlo points and for the total cross-section. The agreement between the full expression for  $C_H$ , Eq. (2), and the approximation, Eq. (5), is excellent for  $\sigma_{\text{tot}}$  ( $\ll 0.1\%$ ), and better than about 0.15% for almost all of phase space. Adding the box amplitudes not proportional to the tree-level  $t$ -channel graph does not significantly change the agreement. However, the other  $m_H$ -dependent graphs are seen to be more important than in the on-shell approximation. Overall, the simple approximation reproduces the full result to about 0.1% in  $\sigma_{\text{tot}}$ , but only 0.4% for individual points in phase space.

For the approximation to hold it is essential that, for a given  $m_W$ , the top-quark mass is varied along with the Higgs-boson mass to nullify any deviations from existing precision data. The values used are given in Table 1 [4]. We note that some values of the top-quark mass are clearly incompatible with the direct measurement [3], but for any value of  $m_H$  there is a range of allowed values of  $m_W$ . For a light Higgs, for instance, there is a preference for the higher  $m_W$  values,

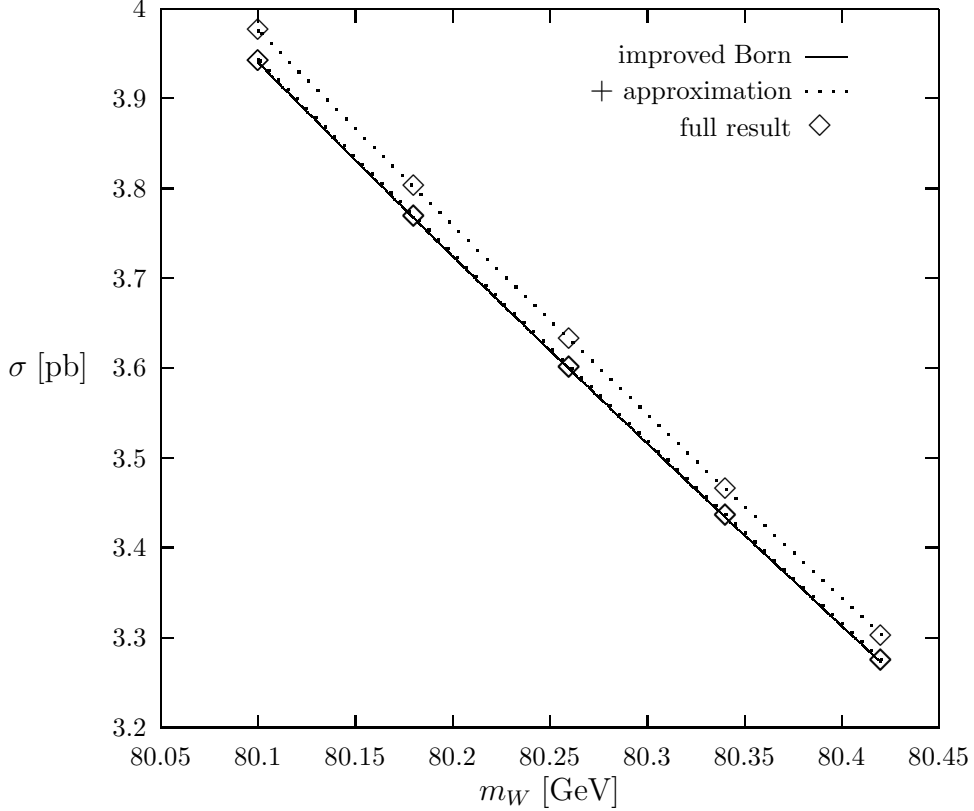


Fig. 3. Total cross-section at  $\sqrt{s} = 161$  GeV as a function of the  $W$ -boson mass for a Higgs-boson mass of 60 and 300 GeV; the latter case yields results that are virtually indistinguishable from the improved-Born curve.

which is supported by the present CDF data [18].

From Table 2 and Figure 3 one can see that the effect of a light Higgs boson ( $m_H = 60$  GeV) is to increase the total cross-section at  $\sqrt{s} = 161$  GeV by about 0.9%. This correction rapidly diminishes for increasing Higgs-boson mass, for  $m_H = 300$  GeV it is negligible ( $< 0.1\%$ ). All this can be translated into an uncertainty on the determination of  $m_W$  from the LEP-2 threshold run. A measured total cross-section will correspond to

$$m_W(m_H = 300 \text{ GeV})^{+15 \text{ MeV}(m_H = 60 \text{ GeV})}_{-0 \text{ MeV}(m_H = 1000 \text{ GeV})} . \quad (6)$$

After the higher-energy LEP-2 runs have taken place, the improved knowledge of  $m_H$  can be used for an *a posteriori* reduction of the  $m_H$ -dependence of the threshold measurement. An increase of the lower bound to  $m_H > 90$  GeV, for instance, would reduce the uncertainty by roughly a factor of two.



## 4 Conclusion

We have investigated one of the possible effects that could influence the total cross-section at the  $W$ -pair threshold at the 1% level: the corrections due to a light Higgs boson. These corrections are in fact slightly smaller (0.8–0.9% depending on  $m_W$ ). The dominant effect comes from the  $t$ -channel box. This effect can be modelled quite easily with the simple approximation Eq. (5), which is accurate to better than 0.1% in the total cross-section (0.4% in some regions of phase space). This simple correction term is available as part of the generator WWF 2.3 [19].

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<ftp://rulgm4.leidenuniv.nl/gj/>.